

## EXTRA-CREDIT HOMEWORK-04/05

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### EXERCISE 1

Consider the following linear program in equational form :

maximize  $4x_1 - 5x_2 - x_3 + 4x_4 + 3x_5$   
subject to

$$\begin{pmatrix} 3 & 2 & 7 & -2 & -5 \\ 6 & -3 & 0 & -4 & -3 \\ 2 & 1 & 4 & 5 & -3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 8 \\ -10 \\ 13 \end{pmatrix}$$

with  $x_1, x_2, x_3, x_4, x_5 \geq 0$ .

Find all the bases of the linear program. For each base, specify whether it is feasible or not, and if it is feasible, specify the feasible solution and the value of the objective function corresponding to it.

The calculations involved are too tedious to be done by hand, so help yourself with a mathematical software package, e.g. Mathematica, Matlab, or Sage (note that Sage can be used interactively online for free). Briefly describe how you used the software in solving the problem.

### EXERCISE 2

Formulate the following problem as a linear program and solve it using a software package. Analyze the solution (e.g., which constraints are satisfied with equality?). Briefly describe how you used the software.

A cargo plane has three compartments, at the front, middle, and rear of the plane. Each compartment has a maximum weight capacity and a maximum volume capacity, as follows :

	Max weight (tons)	Max volume (m <sup>3</sup> )
front	12	420
middle	9	370
rear	14	510

In addition, in order to keep the plane balanced, all three compartments must be loaded to the same percentage of their weight capacity.

Four types of cargo are available for shipping on the next flight. They have the following characteristics :

	Amount available (tons)	Volume (m <sup>3</sup> per ton)
C1	16	60
C2	8	70
C3	10	50
C4	21	45

The profit in shipping each type of cargo is as follows :

	Profit (CHF per ton)
C1	320
C2	460
C3	250
C4	390

How much should we accept from each type of cargo, and how should we distribute each type of cargo among the compartments, in order to maximize the profit of the next flight?