

# DISCRETE OPTIMIZATION EXTRA ASSIGNMENT

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## EXERCISE 1

Consider the following linear program in equational form :

$$\text{Maximize } 10x_1 - 57x_2 - 9x_3 - 24x_4$$

$$\text{subject to } :x_1 - 11x_2 - 5x_3 + 18x_4 + x_5 = 0 \quad (1)$$

$$x_1 - 3x_2 - x_3 + 2x_4 + x_6 = 0 \quad (2)$$

$$x_1 + x_7 = 1 \quad (3)$$

$$x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0 \quad (4)$$

Make a complete diagram of all possible pivot steps of the simplex algorithm running on this LP : Represent each tableau by a circle indicating the basis and other relevant info, and represent each pivot step by an arrow from a circle to another. Differentiate between degenerate and nondegenerate pivot steps. The initial basis is  $\{x_5, x_6, x_7\}$ .

Explain the significance of avoiding cycling in light of this example.

Which path is followed by the algorithm when using Bland's rule ?

## EXERCISE 2

Recall that  $A(n, d)$  denotes the size of the largest possible subset of  $\{0, 1\}^n$  in which every pair of words has Hamming distance at least  $d$ . Calculate the upper bound for  $A(12, 3)$  given by the sphere-packing argument and the one given by the Delsarte LP. Compare them to the true bound for  $A(12, 3)$  given in Andries Brouwer's table at <http://www.win.tue.nl/~aeb/codes/binary-1.html>.