Bachelor semester 4-DO

Discrete Optimization Extra Assignment

EXERCISE 1

Consider the following linear program in equational form :

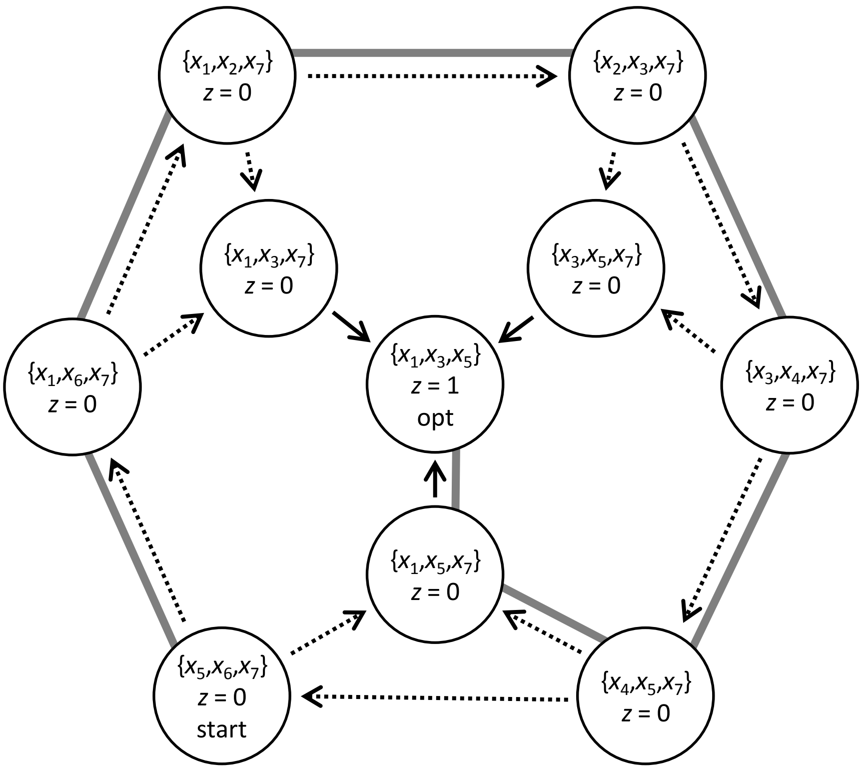
$$\text{Maximize} \quad 10x_1 - 57x_2 - 9x_3 - 24x_4$$

$$\text{subject to}: x_1 - 11x_2 - 5x_3 + 18x_4 + x_5 = 0 \quad (1)$$
$$x_1 - 3x_2 - x_3 + 2x_4 + x_6 = 0 \quad (2)$$
$$x_1 + x_7 = 1 \quad (3)$$
$$x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0 \quad (4)$$

Make a complete diagram of all possible pivot steps of the simplex algorithm running on this LP : Represent each tableau by a circle indicating the basis and other relevant info, and represent each pivot step by an arrow from a circle to another. Differentiate between degenerate and nondegenerate pivot steps. The initial basis is $\{x_5, x_6, x_7\}$.

Explain the significance of avoiding cycling in light of this example.

Which path is followed by the algorithm when using Bland's rule ?



Solution : See attached picture. The dotted arrows are degenerate pivot steps. The solid arrows are non-degenerate pivot steps. The thick gray line indicates the Bland path.

EXERCISE 2

Recall that $A(n,d)$ denotes the size of the largest possible subset of $\{0,1\}^n$ in which every pair of words has Hamming distance at least d. Calculate the upper bound for $A(12,3)$ given by the sphere-packing argument and the one given by the Delsarte LP. Compare them to the true bound for $A(12,3)$ given in Andries Brouwer's table at
http ://www.win.tue.nl/~aeb/codes/binary-1.html.

Solution : The sphere-packing bound gives $A(12,3) <= \lfloor 2^{12}/13 \rfloor = 315$. The Delsarte bound can be computed with Mathematica as follows :

```
K[t_, n_, i_] :=  Sum[(-1)^j Binomial[i, j] Binomial[n - i, t - j], {j, 0, Min[i, t]}]

Delsarte[n_, d_] := Module[
  {c, A, b},
  c = Table[-1, {i, n + 1}];
  A = {{1}~Join~Table[0, {i, n}], {-1}~Join~Table[0, {i, n}]}~Join~
    Table[Table[0, {j, i}]~Join~{1}~Join~Table[0, {j, n - i}], {i,
      d - 1}]~Join~
    Table[Table[0, {j, i}]~Join~{-1}~Join~Table[0, {j, n - i}], {i,
      d - 1}]~Join~Table[Table[K[t, n, i], {i, 0, n}], {t, n}];
  b = {1, -1}~Join~Table[0, {i, 2 (d - 1) + n}];
  Total[LinearProgramming[c, A, b]]
  ]
```

When we call

```
Delsarte[12,3]
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we get 2048/7, so we obtain $A(12,3) <= 292$.

According to Andries Brouwer's table $A(12,3) = A(13,4) = 256$.