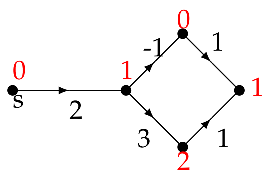
Bachelor semester 4-DO

Discrete Optimization WEEK 10

<u>EXERCISE 1</u>

We are given a directed graph $G = (V, E)$ in which each edge e is assigned a "length" b_e. The lengths may be negative, but we assume that G does not contain a negative cycle (a directed cycle whose total length is negative).
We are also given two vertices s, t in V. We want to find the shortest path from s to t in G. The length of this shortest path is called the "distance" from s to t.

A "feasible potential" γ in G is an assignment of numbers γ_v to the vertices v of V such that :
(1) $\gamma_s = 0$
(2) for every edge uv in E we have $\gamma_v - \gamma_u \leq b_{uv}$.



1) Why is it important to assume that G contains no negative cycle ?
2) Prove that if γ is a feasible potential in G, then γ_t is a lower bound on the distance from s to t.
3) Use the LP duality theorem to prove that there exists a feasible potential γ for which γ_t equals the distance from s to t.
Hint : Formulate the problem of finding the shortest path from s to t as an integer program. Prove that the LP relaxation has an integer solution. Construct the dual of this LP.