ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Bachelor semester 4-DO

DISCRETE OPTIMIZATION ASSIGNMENT 11

EXERCISE 1

Prove the following lemma (known as "complementary slackness") using the duality theorem.

LEMMA

Let $x^* = (x_1^*, ..., x_n^*)$ be a feasible solution of the linear program

$$\begin{aligned} &\text{Maximize } c^T x \\ &\text{subject to : } Ax \leq b, x \geq 0; \end{aligned} \qquad (P)$$

and let $y^* = (y_1^*, ..., y_m^*)$ be a feasible solution of the dual linear program :

$$\begin{aligned} &\text{Minimize } b^T y \\ &\text{subject to : } A^T y \geq c, y \geq 0. \end{aligned} \qquad (D)$$

Then the following two conditions are equivalent :

I) x^* is an optimal solution for (P) and y^* is an optimal solution for (D).

II) For every $i, 1 \leq i \leq m$, if x^* does not satisfy the i-th constraint of (P) with equality then $y_i^* = 0$; and for every j, $1 \leq j \leq n$, if y^* does not satisfy the j-th constraint of (D) with equality then $x_j^* = 0$.

EXERCISE 2

Consider the following LP :

$$\text{Maximize } x_1 + x_2$$

$$\begin{aligned} \text{subject to : } \quad 2x_1 + x_2 &\leq 6 \\ x_1 + 2x_2 &\leq 8 \\ 3x_1 + 4x_2 &\leq 22 \\ x_1 + 5x_2 &\leq 23 \\ x_1, x_2 &\geq 0. \end{aligned}$$

Use complementary slackness to prove that $x^* = (4/3, 10/3)$ is an optimal solution.