ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Bachelor semester 4-

DISCRETE OPTIMIZATION LAST WEEK

EXERCISE 1
(This exercise shows a real-world application of graphs that can have negative lengths.)

We are trading currencies. We are given a set of possible currency exchange rates, where each each rate consists of a pair v, w of currencies, together with a rate r_{vw} that specifies the number of units of currency w that we can purchase for one unit of currency v.
a. Explain how to represent this information as a directed graph, so that finding the best way of converting currency s to currency t corresponds to finding the shortest path from s to t in the graph.
b. What does a negative cycle in the graph represent ?

EXERCISE 2
A directed graph $G = (V, E)$ is "acyclic" if it does not contain any directed cycle. A "topological sort" of a graph is an ordering of its vertices as $v_1, v_2, ..., v_n$ such that every edge goes from left to right.
a. Prove that G is acyclic if and only if it has a topological sort, and provide a linear-time algorithm that either finds a topological sort of G or says that G is cyclic.
(Hint : An acyclic graph must contain a vertex without incoming edges.)
b. Describe a linear-time algorithm that finds all shortest paths from a given vertex s in a given acyclic graph.

EXERCISE 3
Prove Helly's theorem in one dimension, meaning : If $L_1, L_2, ..., L_n$ is a family of intervals of real numbers such that every pair of intervals intersect, then all the intervals intersect (meaning, there exists a real number that is contained in all the intervals).