Bachelor semester 4-DO

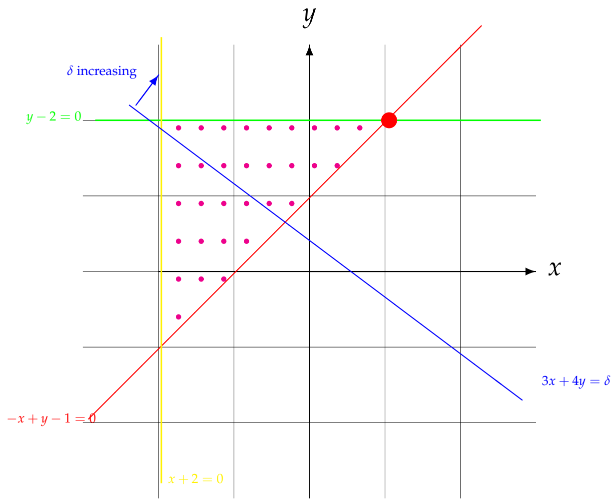
Discrete Optimization WEEK 1

EXERCISE 1

Solve each of the following linear programs by making a diagram. In each case, specify whether the program is feasible and bounded, feasible and unbounded, or unfeasible. If the program is bounded, specify all optimal solutions. If it is unbounded, give an unbounded ray on which the objective function increases without limit.

1. Find the maximal value (if it exists) of $z = 3x + 4y$ subject to the following constraints

$$\begin{cases} x - y + 1 & \leq & 0 \\ \quad y & \leq & 2 \\ -x & \leq & 2 \end{cases}$$



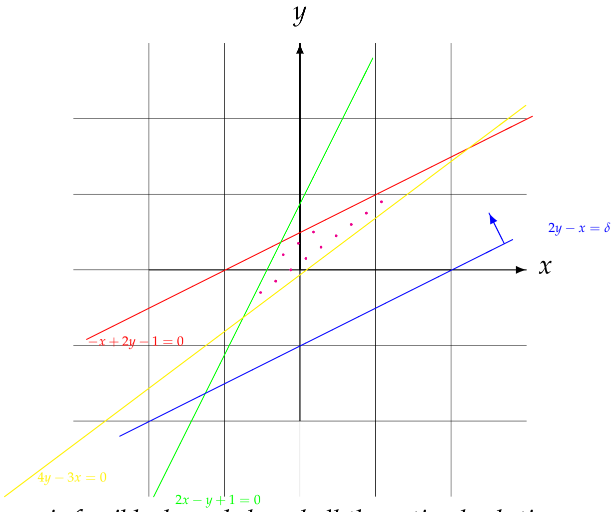
So the program is feasible, bounded, and the optimal solution(s) is/are given by

$$(x, y) = (1, 2)$$

so that $z = 11$.

2. Find the maximal value (if it exists) of $z = 2y - x$ subject to the following constraints

$$\begin{cases} 2y - x - 1 & \leq & 0 \\ y - 2x & \leq & 1 \\ -4y + 3x & \leq & 0 \end{cases}$$



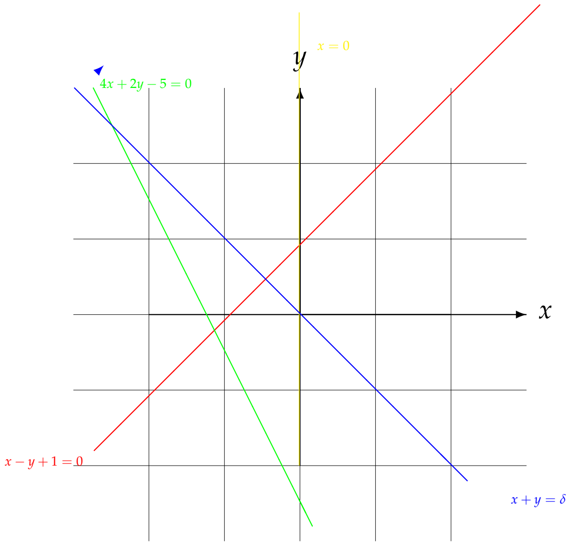
So the program is feasible, bounded, and all the optimal solutions are given by

$$(x, y) = (-1/3, 1/3) + t(7/3, 7/6) \ \ (t \in [0, 1])$$

so that $z = 1$.

3. Find the maximal value (if it exists) of $z = x + y$ subject to the following constraints

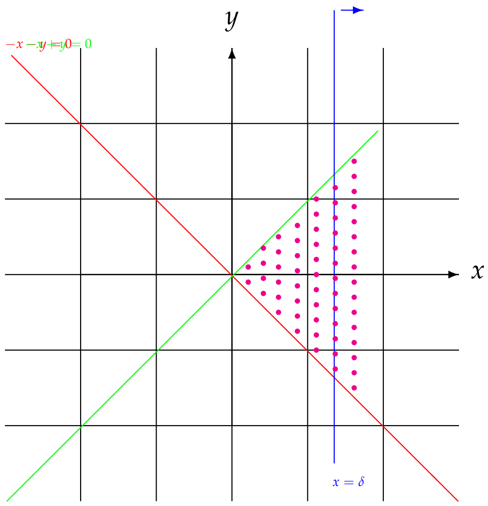
$$\begin{cases} x - y + 1 & \leq & 0 \\ 4x + 2y + 5 & \leq & 0 \\ -x & \leq & 0 \end{cases}$$



So the program is unfeasible.

4. Find the maximal value (if it exists) of $z = x$ subject to the following constraints

$$\begin{cases} -x - y & \leq & 0 \\ -x + y & \leq & 0 \end{cases}$$



So the program is feasible, unbounded, and we see that the points $(x, 0)$ *for* $x > 0$ *are admisible and give z going to infinity.*

5. Find the maximal value (if it exists) of $z = x + y$ subject to the following constraints

$$\left\{ \begin{array}{rcl} x + y - 1 & \leq & 0 \end{array} \right.$$

Here clearly the program is feasible, bounded (although the set of solutions is not) by 1 and of course, the line $y = 1 - x$ *is the set of all optimal solutions.*

<u>EXERCISE 2</u>

A building supply has two locations in town. The office receives orders from two customers, each requiring 3/4-inch plywood.
Customer A needs fifty sheets and Customer B needs seventy sheets.
The warehouse on the east side of town has eighty sheets in stock ; the west-side warehouse has sixty sheets in stock.
Delivery costs per sheet are as follows : 1 CHF from the eastern warehouse to Customer A, 1.10 CHF from the eastern warehouse to Customer B, 0.80 CHF from the western warehouse to Customer A, and 1.05 CHF from the western warehouse to Customer B.
Formulate the Linear Program associated to this problem.

Let Q_{EA} *the quantity shipped to A from the East, so that* $(50 - Q_{EA})$ *is the quantity shipped to A from West. Let* Q_{EB} *the quantity shipped to B from the East, so that* $(70 - Q_{EB})$ *is the quantity shipped to A from West.*
We want to minimize the quantity $C = Cost$ *which is equal to*

$$C = \underbrace{1Q_{EA} + 0.8(50 - Q_{EA})}_{Cost\ for\ A} + \underbrace{1.1Q_{EB} + 1.05(70 - Q_{EB})}_{Cost\ for\ B}$$

with the constraints

$$Q_{EA} + Q_{EB} \leq 80\ AND (50 - Q_{EA}) + (70 - Q_{EB}) \leq 60.$$

Note that finding the min of C is equivalent to find the max of $-C$.