Bachelor semester 4-DO

# DISCRETE OPTIMIZATION WEEK 2

EXERCISE 1

We are given a finite set $S \subseteq \mathbb{R}^2$ of points in the plane, and somebody claims that these points lie appoximately on a circle. How can this be verified ? Here is one possible approach : An *annulus* (ring) is the region between two concentric circles. The *smallest enclosing annulus* of $S$ is the annulus of smallest area that contains $S$ :

If $S$ indeed lies on a circle, then this area is 0, but also if $S$ is "almost" on a circle, the smallest enclosing annulus will form a (small) ring around that circle.

Show that the problem of computing the smallest enclosing annulus of $S \subseteq \mathbb{R}^2$ can be formulated as a linear program !

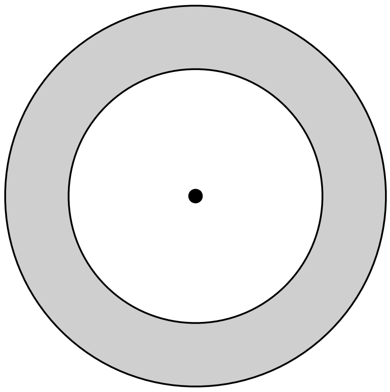

FIG. 1 – An annulus.

EXERCISE 2

Consider the following *greedy algorithm* for finding a vertex cover $S$ in a given graph $G = (V, E)$ : Set $S = \emptyset$. As long as $E$ is nonempty, add to $S$ a vertex $v$ of largest degree (number of incident edges) in $G$. Then delete the edges incident to $v$ from $G$ and repeat the process with the resulting graph $G' = (V, E')$.

Show that there is no constant factor $k$ with the property that this algorithm always finds a vertex cover whose size is at most $k$ times as large as the size of a smallest vertex cover.

EXERCISE 3

A paper mill manufactures rolls of paper of a standard width 4 meters. But customers want to buy paper rolls of shorter width, and the mill has to cut such rolls from the 4 meters rolls.

Consider an order of

- 106 rolls of width 68 cm,
- 28 rolls of width 173 cm,
- 205 rolls of width 148 cm,
- 93 rolls of width 127 cm.

What is the smallest number of 4 m rolls that have to be cut in order to satisfy this order ? Write a linear program where the solution is the answer to this question.