ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Bachelor semester 4-DO

DISCRETE OPTIMIZATION WEEK 3

EXERCISE 1

Consider the following *greedy algorithm* for finding a vertex cover S in a given graph $G = (V, E)$: Set $S = \emptyset$. As long as E is nonempty, add to S a vertex v of largest degree (number of incident edges) in G. Then delete the edges incident to v from G and repeat the process with the resulting graph $G' = (V, E')$.

Show that there is no constant factor k with the property that this algorithm always finds a vertex cover whose size is at most k times as large as the size of a smallest vertex cover.

EXERCISE 2

We are given a set R of m red points and a set B of n blue points in the plane. We want to find a non-vertical line in the plane that passes below all red points and above all blue points (the line is also allowed to pass through points). Write a linear system of inequalities (meaning, a system of the form $A\boldsymbol{x} \leq \boldsymbol{b}$) that expresses whether such a line exists or not.

EXERCISE 3

Consider the following bipartite graph :

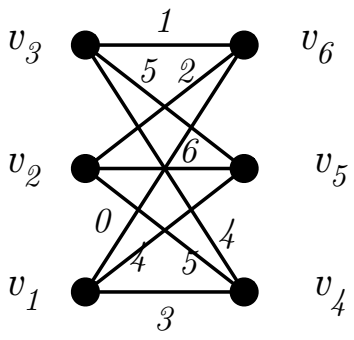

FIG. 1 –

We are interested in finding a maximum-weight perfect matching in this graph.

a) Find a perfect matching of weight 10.

b) Prove that there is no perfect matching of weight larger than 10, as follows : Write the linear program corresponding to the problem, as we learned in class. Multiply the six constraints corresponding to the vertices by 0, 2, 1, 3, 4, and 0, respectively ; add up these constraints ; and interpret the result.

(This is a prelude to the duality theorem that we will learn later in the course.)