DISCRETE OPTIMIZATION WEEK 4

EXERCISE 1

Let p_1, p_2, p_3, p_4 and p_5 be five points in $\mathbb{R}^d$, and let $q \in \mathbb{R}^d$ be the convex combination

$$q = \frac{2}{9}p_1 + \frac{4}{9}p_2 + \frac{1}{9}p_3 + \frac{1}{9}p_4 + \frac{1}{9}p_5$$

Find points r_1, r_2 and r_3 in $\mathbb{R}^d$ such that r_1 lies on the segment p_1p_2, r_2 lies on the segment r_1p_3, r_3 lies on the segment r_2p_4, and q lies on the segment r_3p_5.

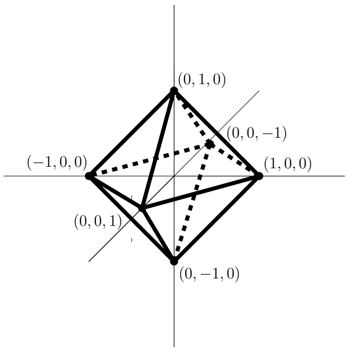
EXERCISE 2

Let $X = [0,1]^3$ be the unit cube in $\mathbb{R}^3$, and suppose we have a linear program whose set of feasible solutions is X. Find an objective function for which :

a) the unique optimal solution is the vertex $(1,0,0)$;
b) the set of optimal solutions is the edge $(1,0,0) - (1,1,0)$;
c) the set of optimal solutions is the face with vertices $(1,0,0), (1,1,0), (1,0,1), (1,1,1)$;
d) the set of optimal solutions is all of X.

EXERCISE 3

a) The n-dimensional crosspolytope is the set $C_n = \{x \in \mathbb{R}^n : |x_1| + |x_2| + ... + |x_n| \leq 1\}$. The following picture shows C_3 :



Express C_n as the solution set of a linear system of inequalities (meaning, a system of the form $Ax \leq b$).

b) Let $D = x \in \mathbb{R}^3 : |2x_1 - x_2 + 3x_3 + 1| + |x_2 + 2x_3 - 2| + |5x_1 - 3x_3| \leq 10$. Express D as the solution set of a linear system of inequalities.

EXERCISE 4

Convert the following linear program to equational form :

Minimize $2x_1 - 3x_2$
Subject to :

$$\begin{aligned} x_1 + 5x_2 &\leq 10 \\ -x_1 + 2x_2 &\geq -5 \\ x_1 &\geq 0 \end{aligned}$$