ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Bachelor semester 4-DO

DISCRETE OPTIMIZATION WEEK 5

EXERCISE 1

Consider an LP that has the following constraints in equational form :

$$\begin{pmatrix} 3 & -1 & 4 & 2 & 1 \\ 0 & 3 & 5 & -6 & 2 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 5 \\ 19 \end{pmatrix}$$

with $x_1, x_2, x_3, x_4, x_5 \geq 0$.

Find the basic feasible solution that has basic variables x_2, x_3 and nonbasic variables x_1, x_4, x_5. Why is this solution indeed feasible ?
Find an objective function for which this is the unique optimal solution.

SOLUTION 1

We solve the system

$$\begin{pmatrix} -1 & 4 \\ 3 & 5 \end{pmatrix} \cdot \begin{pmatrix} x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 5 \\ 19 \end{pmatrix}$$

which has for unique solution $(x_1, x_2) = (3, 2)$.
Since both these values are nonnegative the solution is indeed feasible, and the solution is $x_1 = 0, x_2 = 3, x_3 = 2, x_4 = 0, x_5 = 0$.
It's easy to see that an objective function could be : maximize $-x_1 - x_4 - x_5$.

Exercise 2

Consider the following simplex tableau :

$$\begin{aligned} x_4 &= 1 - x_1 + 4x_2 - x_3 \\ x_5 &= 6 - 2x_2 - x_3 \\ x_6 &= 3 + 2x_1 - x_2 + 2x_3 \\ \hline z &= 10 - x_1 + 3x_2 + x_3 \end{aligned}$$

List all possible choices for the entering and the leaving variable in a pivot step from this tableau. Pick one of these choices and perform the pivot step

Solution 2

The entering variable could be either x_2 or x_3 because making them increase make z increases. If it is x_2 then the leaving variable could be x_5 or x_6(for $x_2 = 3$). If the entering variable is x_3 then the leaving variable is x_4 (for $x_3 = 1$).

Choose for example x_3 as the entering variable, so that the can make it increases until $x_3 = 1$ so that $x_4 = 0$. So computing the pivot step, we find

$$\begin{aligned}
x_3 &= 1 - x_1 + 4x_2 - x_4 \\
x_5 &= 5 + x_1 - 6x_2 + x_4 \\
x_6 &= 5 \quad + 7x_2 - 2x_4 \\
\hline
z &= 11 - 2x_1 + 7x_2 - x_4
\end{aligned}$$