

# DISCRETE OPTIMIZATION WEEK 1

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## EXERCISE 1

State the dual problem  $D_i$  for the initial problem  $P_i$  where

1. ( $P_1$ ) : Max  $2x_1 + 3x_2 - x_3 + x_4$  with constraints

$$\begin{aligned} x_2 + 2x_3 &\leq -1 \\ x_1 + x_2 + x_3 &\leq 0 \\ x_1 - x_2 - 2x_3 &\leq 5 \\ x_2 - x_3 - x_4 &\leq -1 \\ x_i &\geq 0 \quad (i = 1..4) \end{aligned}$$

2. ( $P_2$ ) : Max  $x_1 + x_2 - x_3 + x_4 - 5x_5$  with constraints

$$\begin{aligned} x_i - x_{i+1} &\leq 0 \quad (i = 1..4) \\ x_5 - x_1 &\geq 2 \\ x_1 + x_2 + x_3 + x_4 + x_5 &= 1 \\ x_i &\leq 0 \quad (i = 1..5) \end{aligned}$$

## EXERCISE 2

Consider the following linear program ( $P$ ) : Min  $-2x_1 + x_2$  with constraints

$$\begin{aligned} 2x_1 + 3x_2 &\geq 6 \\ -x_1 + 3x_2 &\geq 6 \\ x_1 &\leq 2 \\ x_i &\geq 0 \end{aligned}$$

1. Write this problem on functional form.
2. Write its dual problem.
3. Use a computer (Scilab for example) to show that this problem and its dual have same solution.

## EXERCISE 3

Imagine, you're a robber in a jewelry, and during a hold-up, as all stupid robbers, you only took one 10 liters backpack with you to bring your booty. In front of you, the jeweler 'offers' you infinite quantities of

- Rings in their beautiful cases, which cost 1000CHF each, with a volume of 1 liter, and a total weight of 100 grams.<sup>1</sup>
- Watches in their smaller case, which cost 1200CHF each<sup>2</sup>, for a volume of 0.2 liter and a total weight of 400 grams.

As you are not the most sportive robber (neither the most intelligent), you can only bring 5kgs with your bag.

We forget now we are looking for solutions in  $\mathbb{Z}$ , so we are looking for real solutions.

As before becoming a robber, you learned optimization at EPFL, you remember your so interesting course in DO. So you can optimize your hold-up.

1. Try to find a linear program describing the situation, and then write it in functional form.
2. Give the dual program of your linear program (not necessarily the one in functional form)
3. Solve it, using a computer.

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1. Yes I know, the case is very large, but you don't have time enough to throw it away.  
2. These are not Roll.. watches.