DISCRETE OPTIMIZATION WEEK 7

EXERCISE 1

State the dual problem D_i for the initial problem P_i where

1. (P_1) : Max $2x_1 + 3x_2 - x_3 + x_4$ with constraints

$$\begin{aligned} x_2 + 2x_3 &\leq -1 \\ x_1 + x_2 + x_3 &\leq 0 \\ x_1 - x_2 - 2x_3 &\leq 5 \\ x_2 - x_3 - x_4 &\leq -1 \\ x_i &\geq 0 \quad (i = 1..4) \end{aligned}$$

2. (P_2) : Max $x_1 + x_2 - x_3 + x_4 - 5x_5$ with constraints

$$\begin{aligned} x_i - x_{i+1} &\leq 0 \quad (i = 1..4) \\ x_5 - x_1 &\geq 2 \\ x_1 + x_2 + x_3 + x_4 + x_5 &= 1 \\ x_i &\leq 0 \quad (i = 1..5) \end{aligned}$$

SOLUTION 1

Using the chart given in class, we have

1. (D_1) : Min $-y_1 + 5y_3 - y_4$ with constraints

$$\begin{aligned} y_2 + y_3 &\geq 2 \\ y_1 + y_2 - y_3 + y_4 &\geq 3 \\ 2y_1 + y_2 - 2y_3 - y_4 &\geq -1 \\ -y_4 &\geq +1 \\ y_i &\geq 0 \quad (i = 1..4) \end{aligned}$$

2. (D_2) : Min $2y_5 + y_6$ with constraints

$$\begin{aligned} y_1 - y_5 + y_6 &\leq 1 \\ -y_1 + y_2 + y_6 &\leq 1 \\ -y_2 + y_3 + y_6 &\leq -1 \\ -y_3 + y_4 + y_6 &\leq 1 \\ -y_4 + y_5 + y_6 &\leq -5 \\ y_1, y_2, y_3, y_4, -y_5 &\geq 0 \\ y_6 &\in \mathbb{R} \end{aligned}$$

EXERCISE 2

Consider the following linear program (P) : Min $-2x_1 + x_2$ with constraints

$$\begin{aligned} 2x_1 + 3x_2 &\geq 6 \\ -x_1 + 3x_2 &\geq 6 \\ x_1 &\leq 2 \\ x_i &\geq 0 \end{aligned}$$

1. Write this problem on equational form.
2. Write its dual problem.

3. Use a computer (Scilab for example) to show that this problem and its dual have same solution.

SOLUTION 2

1. The equational form of the problem is (P) : Max $2x_1 - x_2$ with constraints

$$\begin{aligned} 2x_1 + 3x_2 - x_3 &= 6 \\ -x_1 + 3x_2 - x_4 &= 6 \\ x_1 - x_5 &= 2 \\ x_i &\geq 0 \end{aligned}$$

Note that the solution of the initial problem will be - the solution found (because $Minf = -Max(-f)$)

2. The dual problem is (P) : Min $6y_1 + 6y_2 + 2y_3$ with constraints

$$\begin{aligned} 2y_1 - y_2 + y_3 &\geq 2 \\ -x_1 + 3x_2 - x_4 &\geq -1 \\ -y_1 &\geq 0 \\ -y_2 &\geq 0 \\ -y_3 &\geq 0 \\ y_i &\in \mathbb{R} \end{aligned}$$

This means (P) : Min $6y_1 + 6y_2 + 2y_3$ with constraints

$$\begin{aligned} 2y_1 - y_2 + y_3 &\geq 2 \\ -x_1 + 3x_2 - x_4 &\geq -1 \\ y_1, y_2, y_3 &\leq 0 \end{aligned}$$

3. We just give the scilab code to solve the initial problem :
–>p=[-2 ;1 ;0 ;0 ;0] ;b=[6 ;6 ;2] ;Q=[0,0,0,0,0 ;0,0,0,0,0 ;0,0,0,0,0 ;0,0,0,0,0 ;0,0,0,0,0] ;

–> ci=[0 ;0 ;0 ;0 ;0] ;cs=[10^9; 10^9; 10^9; 10^9; 10^9]; $C = [2, 3, -1, 0, 0; -1, 3, 0, -1, 0; 1, 0, 0, 0, 1]; me = 3;$

–>[x,lagr]=qld(Q,p,C,b,ci,cs,me) ;

EXERCISE 3

Imagine, you're a robber in a jewelry, and during a hold p-up, as all stupid robbers, you only took one 10 liters backpack with you to bring your booty. In front of you, the jeweler 'offers' you infinite quantities of

- Rings in their beautiful cases, which cost 1000CHF each, with volume of 1 liter, and a total weight of 100 grams. [1]
- Watches in their smaller case, which cost 1200CHF each [2], for a volume of 0.2 liter and a total weight of 400 grams.

As you are not the most sportive robber (neither the most intelligent), you can only bring 5kgs with your bag.
We forget now we are looking for solutions in $\mathbb{Z}$, so we are looking for real solutions.
As before becoming a robber, you learned optimization at EPFL, you remember your so interesting course in DO. So you can optimize your hold-up.

1. Try to find a linear program describing the situation, and then write it in equational form.
2. Give the dual program of your linear program (not necessary the one in equational form)
3. Solve it, using a computer.

SOLUTION 3

(a) We see that the problem to solve is (P) : Max $1000x_1 + 1200x_2$ with constraints

$$\begin{aligned} 10x_1 + 2x_2 &\geq 10 \\ x_1 + 4x_2 &\geq 50 \\ x_i &\geq 0 \end{aligned}$$

for which the equational form is (P) : Max $1000x_1 + 1200x_2$ with constraints

$$\begin{aligned} 10x_1 + 2x_2 - x_3 &= 10 \\ x_1 + 4x_2 - x_4 &= 50 \\ x_i &\geq 0 \end{aligned}$$

1. Yes I know, the case is very large, but you don't have time enough to throw it away.
2. These are not Roll.. watches.

(b) The dual problem is (P) : Min $100y_1 + 50y_2$ with constraints

$$\begin{aligned} 10y_1 + y_2 &\geq 1000 \\ 2y_1 + 4y_2 &\geq 1200 \\ y_i &\geq 0 \end{aligned}$$

(c) We give the Scilab code to solve the initial problem

–>p=[-1000 ;-1200] ;b=[100 ;50] ;Q=[0,0 ;0,0] ; ci=[0 ;0] ;cs=[$10^9; 10^9$]; $C = [10, 2; 1, 4]; me = 0;$

–>[x,lagr]=qld(Q,p,C,b,ci,cs,me) ;

Notice that we put $p = [-1000; -1200]$ because the function qld deals with inequalities in the other sense. Note that QLD is a minimizing function.

The solution is $(x_1 = 7.8..., x_2 = 10.5...)$ so that we are sure that we can at least take 7 rings and 10 watches, and that this is not a so bad solution (but it's not sure to be the best one in integers).