

## DISCRETE OPTIMIZATION WEEK 8

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### EXERCISE 1

Let  $A$  is a matrix with  $m$  rows and  $n$  columns and  $\mathbf{b} \in \mathbb{R}^m$ . For each of the following statements fill in the blank.

- a) If the system  $A\mathbf{x} = \mathbf{b}, \mathbf{x} \geq 0$  is infeasible then there exists a  $\mathbf{y} \in \mathbb{R}^m$  such that \_\_\_\_\_.
- b) If the system  $A\mathbf{x} \leq \mathbf{b}, \mathbf{x} \geq 0$  is infeasible then there exists a  $\mathbf{y} \in \mathbb{R}^m$  such that \_\_\_\_\_.
- c) If the system  $A\mathbf{x} \leq \mathbf{b}, \mathbf{x} \in \mathbb{R}^n$  is infeasible then there exists a  $\mathbf{y} \in \mathbb{R}^m$  such that \_\_\_\_\_.
- d) If the system  $A\mathbf{x} = \mathbf{b}, \mathbf{x} \in \mathbb{R}^n$  is infeasible then there exists a  $\mathbf{y} \in \mathbb{R}^m$  such that \_\_\_\_\_.

Which of the statements is weaker than the other?

### EXERCISE 2

For the following set of inequalities, do Fourier-Motzkin elimination on the variable  $x$ . Afterwards, show how one can view the new set of inequalities as a linear combination of the original ones, that is, give the numbers for which you could multiply the original inequalities in order to obtain the new set of inequalities.

$$\begin{aligned}4x + 5y &\leq 12 \\3x - 9y &\leq 3 \\-2x + 5y &\leq -10 \\7x + 3y &\leq -14 \\-x + 2y &\leq 2\end{aligned}$$