ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Bachelor semester 4-DO

DISCRETE OPTIMIZATION WEEK 8

SOLUTIONS 1

Let A is a matrix with m rows and n columns and $\mathbf{b} \in \mathbb{R}^m$. For each of the following statements fill in the blank.

a) If the system $A\mathbf{x} = \mathbf{b}, \mathbf{x} \geq 0$ is infeasible then there exists a $\mathbf{y} \in \mathbb{R}^m$ such that $\underline{\mathbf{y}^T A \geq 0^T \text{ and } \mathbf{y}^T \mathbf{b} < 0}$.

b)If the system $A\mathbf{x} \leq \mathbf{b}, \mathbf{x} \geq 0$ is infeasible then there exists a $\mathbf{y} \in \mathbb{R}^m$ such that $\underline{\mathbf{y} \geq 0, \mathbf{y}^T A \geq 0^T \text{ and } \mathbf{y}^T \mathbf{b} < 0}$.

c)If the system $A\mathbf{x} \leq \mathbf{b}, \mathbf{x} \in \mathbb{R}^n$ is infeasible then there exists a $\mathbf{y} \in \mathbb{R}^m$ such that $\underline{\mathbf{y} \geq 0, \mathbf{y}^T A = 0^T \text{ and } \mathbf{y}^T \mathbf{b} < 0}$.

d) If the system $A\mathbf{x} = \mathbf{b}, \mathbf{x} \in \mathbb{R}^n$ is infeasible then there exists a $\mathbf{y} \in \mathbb{R}^m$ such that $\underline{\mathbf{y}^T A = 0^T \text{ and } \mathbf{y}^T \mathbf{b} \neq 0}$.

Which of the statements if weaker than the other ?

EXERCISE 2

For the following set of inequalities, do Fourier-Motzkin elimination on the variable x. Afterwise, show how one can view the new set of inequalities as a linear combination of the original ones, that is, give the numbers for which you could multiply the original inequalities in order to obtain the new set of inequalities.

$$4x + 5y \leq 12 \tag{1}$$
$$3x - 9y \leq 3 \tag{2}$$
$$-2x + 5y \leq -10 \tag{3}$$
$$7x + 3y \leq -14 \tag{4}$$
$$-x + 2y \leq 2 \tag{5}$$

SOLUTIONS 2

On the first step, we get the following inequalities.

$$x \leq 3 - \frac{5}{4}y$$
$$x \leq 1 + 3y$$
$$x \geq 5 + \frac{5}{2}y$$
$$x \leq -2 - \frac{3}{7}y$$
$$x \geq 2y - 2$$

These are then transformed into the following inequalities.

$$3 - \frac{5}{4}y \geq 5 + \frac{5}{2}y$$
$$3 - \frac{5}{4}y \geq 2y - 2$$
$$1 + 3y \geq 5 + \frac{5}{2}y$$
$$1 + 3y \geq 2y - 2$$
$$-2 - \frac{3}{7}y \geq 5 + \frac{5}{2}y$$
$$-2 - \frac{3}{7}y \geq 2y - 2$$

Rewriting :

$$15y \leq -8 \tag{6}$$
$$13y \leq 20 \tag{7}$$
$$-y \leq -8 \tag{8}$$
$$-y \leq 3 \tag{9}$$
$$41y \leq -98 \tag{10}$$
$$17y \leq 0 \tag{11}$$

- equation 6= equation 1+ 2×equation 3.
- equation 7= equation 1 + 4×equation 5.
- equation 8= $\frac{2}{3}$×equation 2+ $\frac{3}{2}$×equation 3.
- equation 9=$\frac{1}{3}$×equation 2+ equation 5.
- equation 11=2×equation 4+ 7×equation 3.
- equation 11=equation 4+ 7×equation 5.