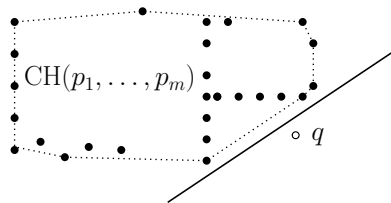


DISCRETE OPTIMIZATION WEEK 9

EXERCISE 1

Let $\mathbf{p}_1, \dots, \mathbf{p}_m, \mathbf{q}$ be points in \mathbb{R}^n . Prove that if $\mathbf{q} \notin \text{CH}(\mathbf{p}_1, \dots, \mathbf{p}_m)$ then there exists a hyperplane that separates \mathbf{q} from $\mathbf{p}_1, \dots, \mathbf{p}_m$.



Hint : Write a system S of linear equalities and inequalities such that S has a solution if and only if $\mathbf{q} \in \text{CH}(\mathbf{p}_1, \dots, \mathbf{p}_m)$. Apply Farkas's lemma to S , and interpret the result geometrically.