ARIEL UNIVERSITY

About the course

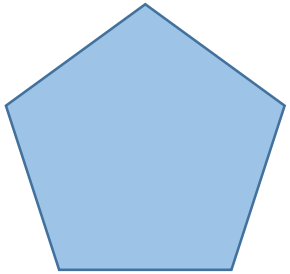
“Discrete Geometry”

Gabriel Nivasch

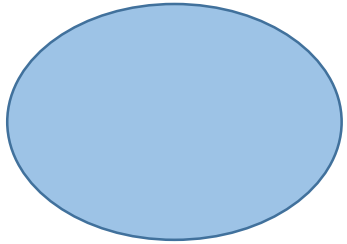
September 4, 2016

Selected topics from the course

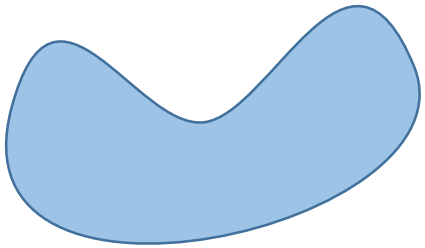
Convex set: A point set *P* is ***convex*** if, for every two points *a*, *b* in *P*, the line segment *ab* is entirely contained in *P*.



convex

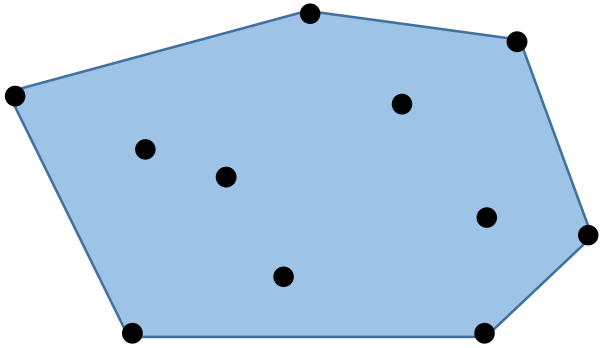


convex

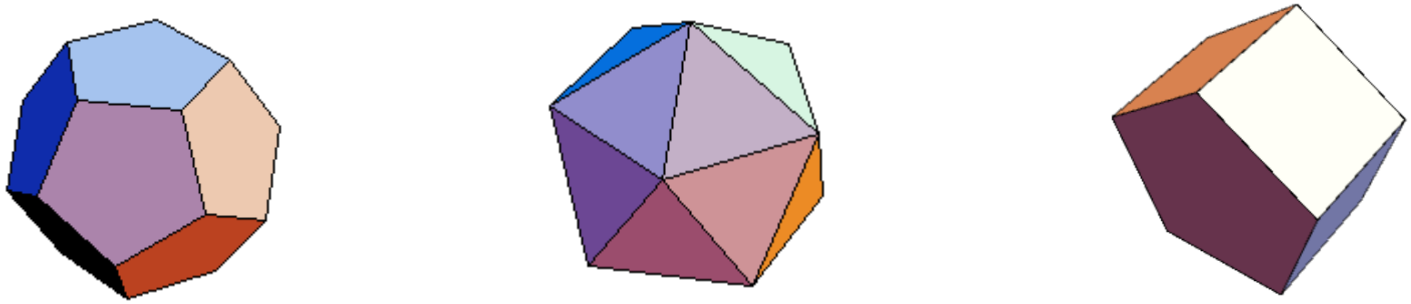


not convex

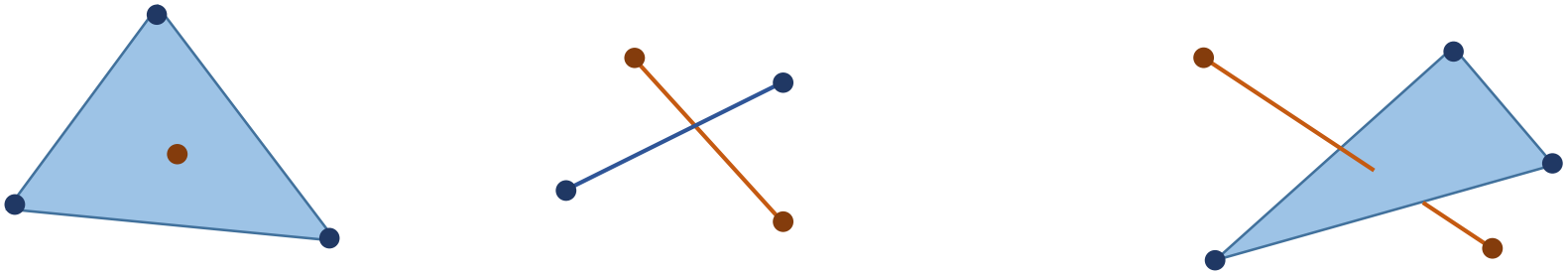
Convex hull: The minimal convex set that contains the given points.



Convex sets in $\mathbf{R}^3$

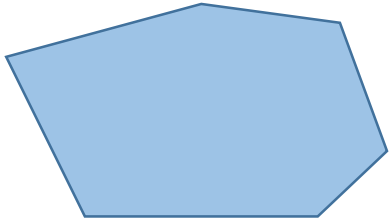


Radon's lemma: A set of d+2 points in $\mathbf{R}^d$ can always be partitioned into two subsets whose convex hulls intersect.

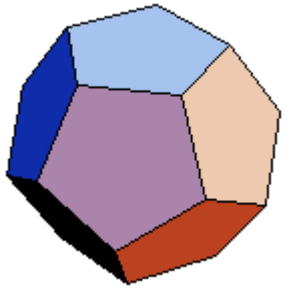


$d = 2$ $d = 2$ $d = 3$

Convex polytopes:

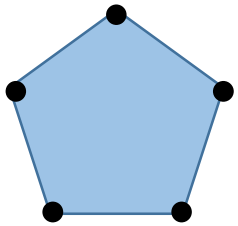


in $\mathbf{R}^2$



in $\mathbf{R}^3$

Definition 1: The convex hull of finitely many points.

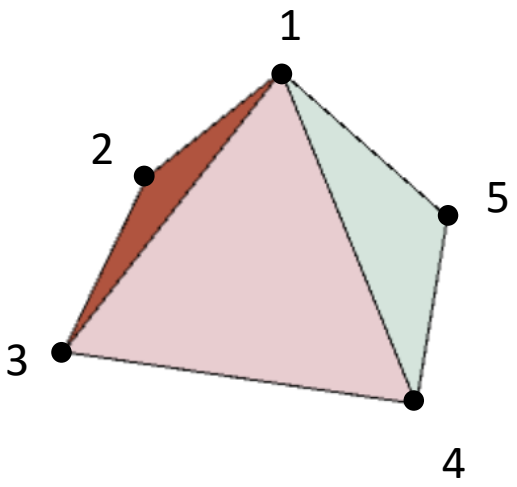


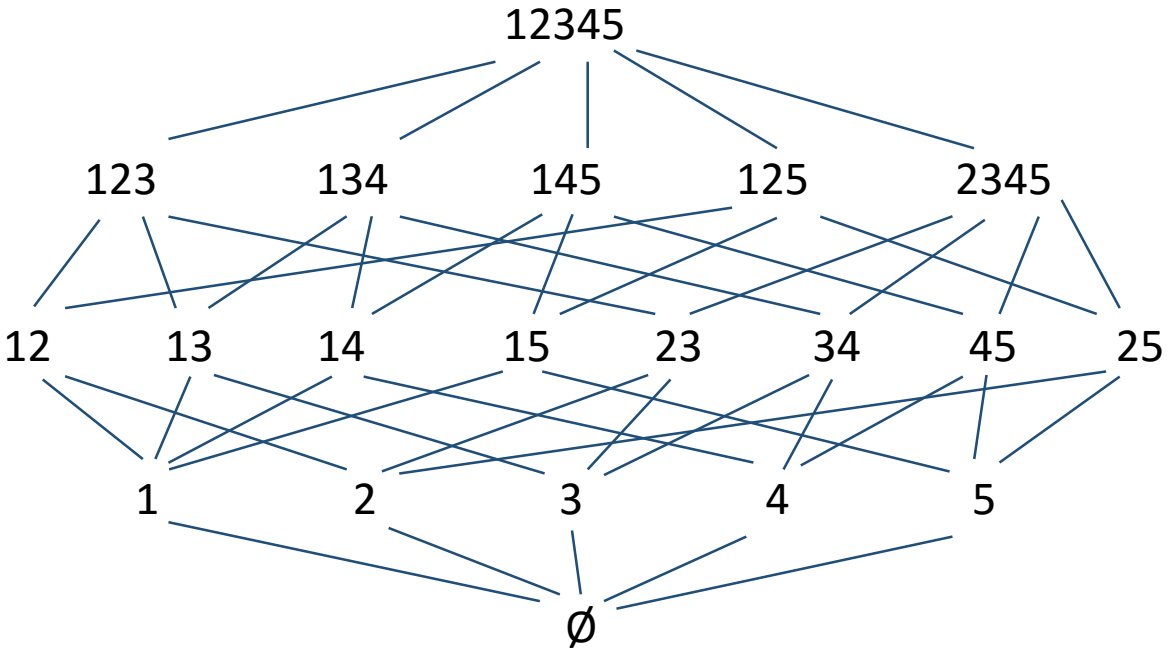
Definition 2: The intersection of finitely many halfspaces.

Theorem: These two definitions are equivalent.

The faces of a polytope are lower-dimensional polytopes

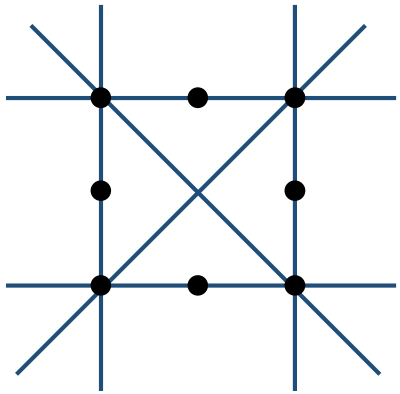
Face lattice of a square pyramid:





Incidences between points and lines

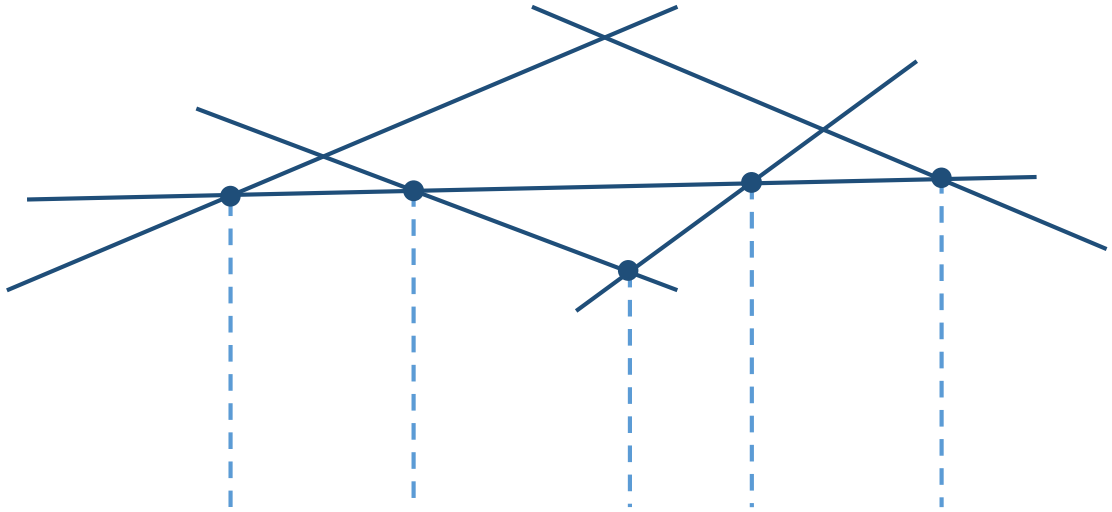
Given integers *m* and *n*, arrange *m* points and *n* lines so as to maximize the number of ***incidences*** between points and lines.



8 point
6 lines
16 incidences

Lower envelopes

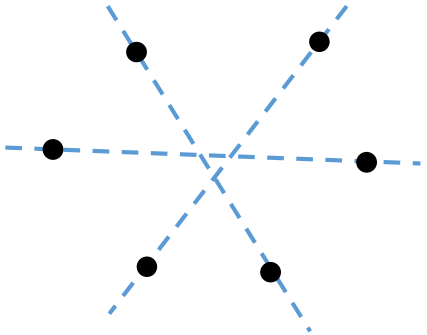
Given an integer *n*, arrange *n* line segments so that as many intersections as possible are visible from infinity below



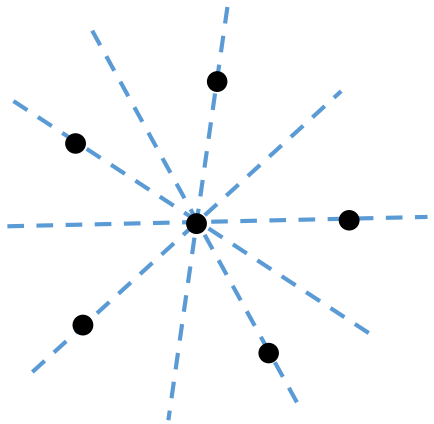
Halving lines

Let n be an even integer. Let P be a set of n points (no three on a line). A ***halving line*** is a line through two points of P that splits all the remaining points into two equal parts.

$n = 6$:



3 halving lines

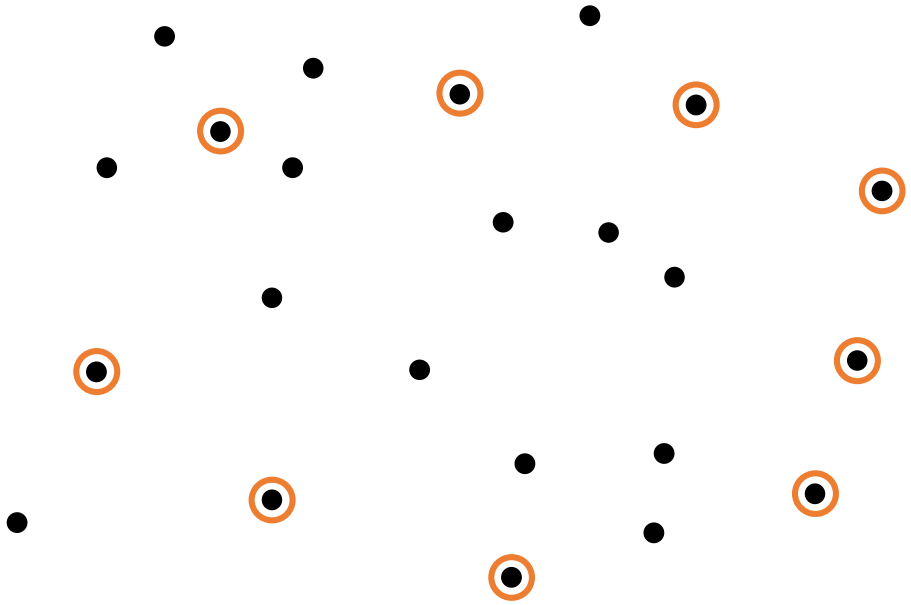


5 halving lines

Given n, arrange n points so as to maximize the number of halving lines.

Points in convex position

Given an integer k, how large must n be so that, no matter how we arrange n points in the plane (no three on a line), there will always be k points in ***convex position***?

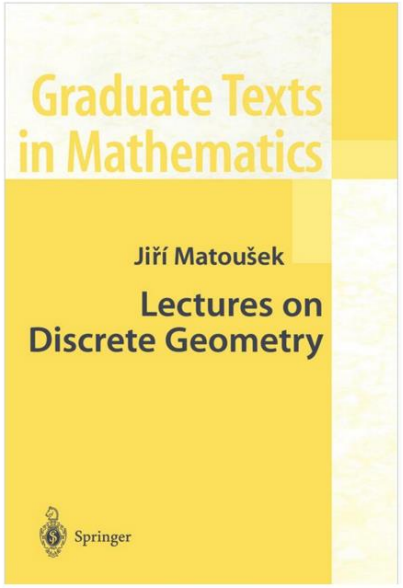


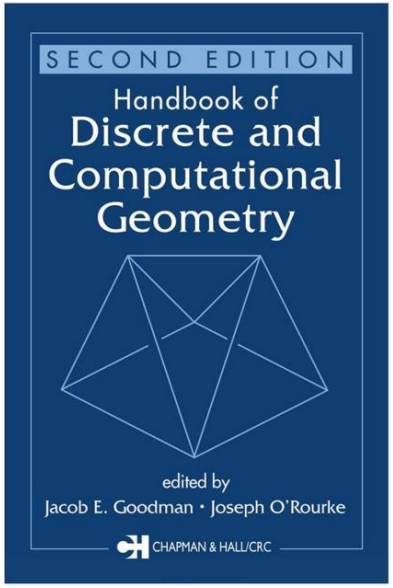
$k = 9$

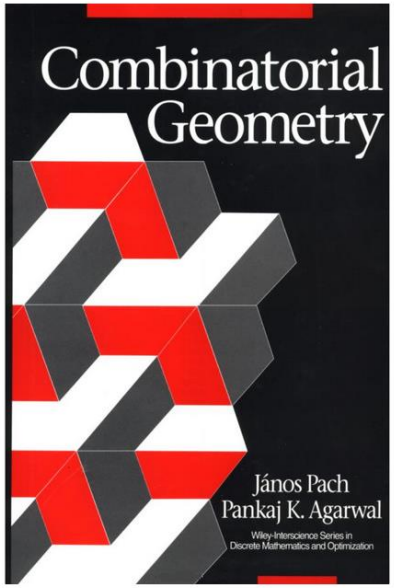
About the course

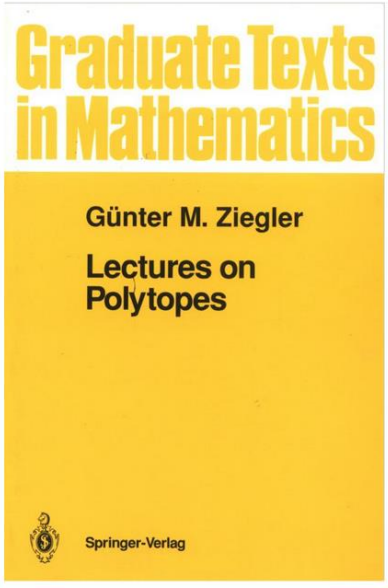
Grading: Homeworks: 20% Final exam: 80%

Selected bibliography:









Contact: Gabriel Nivasch gabrieln@ariel.ac.il