

## תרגילי קומבינטוריקה – פתרונות חלקיים

.2

$$\binom{7}{2} = 21 \quad \text{א.}$$

$$10! 2^3 3^4 5^3 / (3! 4! 3!) = 340,200,000 \quad \text{ב.}$$

.4

$$4^6 \binom{9}{3} = 344,064 \quad \text{א.}$$

$$2^9 \binom{5}{2} - 3 \cdot 5 = 5,105 \quad \text{ב.}$$

$$5 \cdot 4^8 = 327,680 \quad \text{ג.}$$

$$5^5 = 5,125 \quad \text{ד.}$$

$$10! / (2!^5) = 113,400 \quad \text{ה.}$$

.5

$$n!/2 \quad \text{א.}$$

$$2(n-1)! \quad \text{ב.}$$

$$n! - 2(n-1)! \quad \text{ג.}$$

$$3^5(2^5 - 1)(3^5 - 2)(3^5 - 3) = 3,401,339,040 \quad \text{.6}$$

$$\binom{11}{4} = 330 \quad \text{.8}$$

$$3^n \quad \text{.9}$$

$$\binom{54}{4} - 5 \binom{33}{4} + \binom{5}{2} \binom{12}{4} = 116,601 \quad \text{.10}$$

.11

$$26! - 24! - 23! - 22! + 21! + 20! + 19! - 17! = \quad \text{א.}$$

$$402,644,090,352,539,326,427,136,000$$

$$26! - 3 \cdot 24! + 2 \cdot 22! = 401,432,363,922,861,472,481,280,000 \quad \text{ב.}$$

.12

$$10,000 - \left\lfloor \frac{10,000}{5} \right\rfloor - \left\lfloor \frac{10,000}{6} \right\rfloor - \left\lfloor \frac{10,000}{7} \right\rfloor + \left\lfloor \frac{10,000}{30} \right\rfloor + \left\lfloor \frac{10,000}{35} \right\rfloor + \left\lfloor \frac{10,000}{42} \right\rfloor - \left\lfloor \frac{10,000}{210} \right\rfloor = \quad \text{א.}$$

$$5,715$$

$$10,000 - \left\lfloor \frac{10,000}{10} \right\rfloor - \left\lfloor \frac{10,000}{12} \right\rfloor - \left\lfloor \frac{10,000}{15} \right\rfloor + \left\lfloor \frac{10,000}{60} \right\rfloor + \left\lfloor \frac{10,000}{30} \right\rfloor + \left\lfloor \frac{10,000}{60} \right\rfloor - \left\lfloor \frac{10,000}{60} \right\rfloor = \quad \text{ב.}$$

$$8,000$$

.13

$$10! \sum_{i=0}^n \frac{(-1)^i}{i!} = 1,334,961$$

.14

א. נוסחת נסיגה:  $x_0 = 1, x_1 = 2, x_n = x_{n-1} + x_{n-2}$   
 פתרון:  $x_n = \frac{5+3\sqrt{5}}{10} \left(\frac{1+\sqrt{5}}{2}\right)^n + \frac{5-3\sqrt{5}}{10} \left(\frac{1-\sqrt{5}}{2}\right)^n$   
 ב.  $n + 1$   
 ג.  $x_0 = 1, x_1 = 2, x_2 = 4, x_n = x_{n-1} + x_{n-2} + x_{n-3}$   
 ד.  $x_0 = 1, x_1 = 2, x_2 = 4, x_n = 2x_{n-1} - x_{n-2} + x_{n-3}$

.15

א.  $4 \cdot 3^n$   
 ב. נוסחת נסיגה:  $x_0 = 1, x_1 = 4, x_n = 4x_{n-1} - x_{n-2}$   
 פתרון:  $x_n = \frac{3+2\sqrt{3}}{6} (2 + \sqrt{3})^n + \frac{3-2\sqrt{3}}{6} (2 - \sqrt{3})^n$   
 ג. נוסחת נסיגה: (\*)  $x_1 = 4, x_2 = 16, x_n = 3x_{n-1} + 3x_{n-2}$   
 פתרון:  $x_n = \frac{14+2\sqrt{21}}{21} \left(\frac{3+\sqrt{21}}{2}\right)^n + \frac{14-2\sqrt{21}}{21} \left(\frac{3-\sqrt{21}}{2}\right)^n$

.16

א. נוסחת נסיגה:  $x_0 = 1, x_1 = 1, x_n = x_{n-1} + x_{n-2}$   
 פתרון:  $x_n = \frac{5+\sqrt{5}}{10} \left(\frac{1+\sqrt{5}}{2}\right)^n + \frac{5-\sqrt{5}}{10} \left(\frac{1-\sqrt{5}}{2}\right)^n$   
 ב.  $x_0 = 1, x_1 = 1, x_2 = 2, x_n = x_{n-1} + x_{n-2} + x_{n-3}$   
 ג.  $x_0 = 1, x_1 = 0, x_2 = 1, x_3 = 0, x_4 = 1, x_n = x_{n-2} + x_{n-5}$

17. נוסחת נסיגה:  $x_0 = 1, x_1 = 2, x_2 = 8, x_n = 3x_{n-1} + 2x_{n-2} - 2x_{n-3}$

פתרון:  $x_n = \frac{2}{7}(-1)^n + \frac{5+3\sqrt{2}}{14} (2 + \sqrt{2})^n + \frac{5-3\sqrt{2}}{14} (2 - \sqrt{2})^n$

(\*) כאן נוסחת הנסיגה מתחילה מ-  $n = 1$ , כי  $n = 0$  הוא מקרה חריג.